Mathematics and the aesthetic experiences it provides

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Does mathematics have value independent of the benefits it provides to humans? In this essay, I argue that mathematics has final (i.e. we value mathematics for its own sake) and intrinsic (i.e. the value mathematics has is not derived from anything else) value. I borrow Korsgaard's terminology from her two distinctions of goodness: intrinsic/extrinsic, and final/instrumental. More specifically, I argue that mathematics' final value exists, and is evident in the aesthetic experiences that mathematics provides to us. Thus, I will also argue that mathematics provides aesthetic experiences, if one attends to mathematics for its own sake. A precise description of mathematics' final and intrinsic value and the aesthetic experiences it provides will remain an open question. This vagueness is a consequence of the many open questions in aesthetics, e.g. whether aesthetic properties exist. Thus, I aim to supply a nonconstructive "proof" which simply argues for the existence of such a final and intrinsic value, without specifying exactly what this value is. Some might object to my claim, arguing that aesthetic experiences are observer-dependent, and therefore necessarily fails to show that mathematics has final value, as such a value necessitates the participation of observers for said value to arise. I will defend my claim by arguing that mathematics passes the isolation test, as its capacity to provide aesthetic experiences persists even in a world where there are no observers to perceive anything at all. To begin, I will define the key terms involved in my arguments, such as Korsgaard's two distinctions of goodness, Stecker's aesthetic experiences, and Shapiro's structuralism. Then, I will construct my argument—that aesthetic experiences arise from mathematics. Thirdly, I will respond to common objections, specifically to those who deny that mathematics has no final or intrinsic value, and to the concerns of whether we can perceive mathematics in the first place. To close, I will discuss the limitations of my arguments, e.g. that our best frameworks about the value of objects is unable to give a more precise and rigorous description of an object's final value, and acknowledge that my argument succeeds nonetheless (argument form adapted from Lo, 2025a).

Korsgaard (1983) argues for two distinctions of goodness: intrinsic/extrinsic goodness, and final/instrumental goodness (p. 169). Korsgaard argues that if an object is valued "for its own sake", the object has final value (p. 170). Likewise, if an object is valued "for the sake of something else", then it is instrumentally good (p. 170). This is her final/instrumental goodness distinction. In contrast, her intrinsic/extrinsic value goodness is about where the object's value resides. If the object is valuable by itself (i.e. in isolation), then it is intrinsically good (p. 170). If the object derives value from "some other source", it is extrinsically good (p. 170). Thus, while some appropriately equate the two, these distinctions provide the basis of my argument (adapted from Lo, 2025a).

Stecker (2019) defines aesthetic experiences as "the experience of attending in a discriminating manner to forms, qualities or meaningful features of things, attending to these for their own sake or for the sake of this very experience" (p. 23). Stecker then argues that we need not decide whether there is a correct or objective definition of aesthetic value (p. 20). He argues that we can choose a definition which suits our project, and that we should be aware of the limitations of our choice definition and our act of choosing. He notes that aesthetic value can be "the value of a type of experience" (p. 21), which is subjective (p. 22), as someone is required to value the experience before the experience itself has aesthetic value. He also notes that "[a]esthetic value is a 'first order' value that is not constituted by other values" (p. 17; adapted from Lo, 2025a).

Shapiro (2000) argues that we should adopt the position of structuralism in mathematics, i.e. believe that mathematics is the science of structures (p. 259). He defines a system as "a collection of objects with certain relations among them" (p. 259). He then defines a structure as the "abstract form of a system, highlighting the interrelationships among the objects" (p. 259). In particular, one disregards any non-essential features (i.e. the features that have no influence on relations) about systems when abstracting to a structure. There is some debate about the commitments a structuralist should adopt. Eliminative structuralists argue that we should not, and need not be, committed to the existence of mathematical structures, as we can appeal to some (fundamental) background ontology which gives rise to mathematical structures (p. 271). On the other hand, noneliminative structuralists argue that we should be committed to the existence of mathematical structures (p. 161). Dedekind argues that such a system exists—that a simply infinite system exists. Dedekind defines "a set S and function s to be a “simply infinite system” if s is one-to-one, there is an element e of S such that e is not in the range of s (thus making S Dedekind-infinite), and the only subset of S that both contains e and is closed under s is S itself. In effect, a simply infinite system is a model of the natural numbers." (Shapiro, 1997, p. 176; adapted from Lo, 2025b).

The debate about the ontology of mathematics is rich and detailed. The point I want to make by adopting Shapiro's structuralism is to argue that mathematics is about something, and that philosophers and mathematicians mean something precise and rigorous when one discusses and practices mathematics. I argue that most mathematicians, when pressed, would adopt structuralism, as set theory is about mathematical structures, e.g. $\mathscr{N} = (\mathbb{N}, (f_i)_{i\leq n}, (R_j)_{i\leq n}, (c_k)_{i\leq n})$, with some language $\mathscr{L} = ((f_i)_{i\leq n}, (R_j)_{i\leq n}, (c_k)_{i\leq n}, =, <)$. Thus, one could say that all other branches of mathematics are reducible to set theory, making all of mathematics about structures—a type of mathematical entity which enjoys an uncontroversial status in mathematics. Suppose one disagrees that set theory is fundamental in this sense. As such, they would rather we take e.g. category theory to be fundamental. This is fine. One simply adopts that ontology instead. My point here is that there is an ontology of mathematics. I adopt structuralism because I believe in it. You need not agree with me. My main argument form will succeed even if you replace my premise of structuralism with some other ontology. But I am getting ahead of myself. Recall Stecker's definition of aesthetic experiences. Note that one needs to attend to something. This is a necessary condition for aesthetic experiences to arise. Thus, if an object does not exist, it will never provide aesthetic experiences, as we are unable to attend to that which does not exist. Thus, I would like to outright deny the objection that we are unable to attend to mathematics. This is before we even discuss whether we are attending to something for its own sake or for the sake of something else. This objection rests on the denial of mathematics' existence. One would need to argue that mathematics is just some convention, social or otherwise. One would then argue that mathematics has no final or intrinsic value, as it is simply a convention, or a collection of social interactions, or something of that sort. I disagree with this objection. I argue that mathematics' existence is independent of us.

Consider any mathematical statement. Would this statement still be true, even if all humans, or all life, did not exist? In the canon of philosophy of mathematics, some argue that mathematical knowledge has three components: it is necessarily true, it is a priori, and it is abstract (Linnebo, 2017, p. 3; adapted from Lo, 2025b). For the purpose of this essay, I remain silent on whether mathematics is a priori. It is self-evident that mathematics is abstract, and necessarily true. Thus, if one accepts that mathematics is abstract and necessarily true, it would be strange if one then asserts that mathematics' existence is dependent on us. Thus, mathematics independently exists from us.

Given that mathematics exists, that it is about structures, and it is abstract, can we attend to it? I pose this question, disregarding whether we are attending to it for its own sake, or for the sake of anything else. The concern is whether we can attend to abstract objects. One could argue that we fail to attend to mathematics, and instead are attending to e.g. our mental states. The concern, stated differently, is how can we perceive that which is abstract? I would like to appeal to classical music as an example where we do attend to something that is abstract, but I recognise the difficulties this appeal faces. For example, when we attend a concert, and an orchestra plays a concerto, what exactly are we attending to? The easy way out is to say we are attending to the performance itself. But this says nothing about the concerto. Are we attending to the concerto by attending to the performance? It is hard to say. We would need to decide on what a piece of music is, and how it is instantiated in the performance. If one argues that concertos are nothing more than specific vibrations of the air (i.e. the sounds we hear), brought about by an orchestra who reads notes off of music sheets, then the answer is decidedly simple. Thus, we perceive the concerto by simply listening to it, and this act of listening is uncontroversial as it is a physical process, well understood in physics. But there is no consensus as to what a piece of classical music is, or whether it is instantiated in the performance of an orchestra, or its sheet music, &c. This example runs into the many open problems in the philosophy of art and aesthetics. These questions carry over to our discussion of mathematics. When we read a journal article, or a proof on a whiteboard, are we attending to the proof itself, or are we attending to the journal article, or the whiteboard with some symbols on it? Mathematics faces additional worries, as it is a social practice. If we were in the classroom, and we were discussing the axiom of choice, are we attending to the class itself—the social engagement, or the axiom of choice itself? A natural response is why not both? Why would we not be attending to the class and the axiom simultaneously? Why would we not be attending to the performance and the concerto simultaneously? The concern here is not about motivation. It is the metaphysical question of whether we are able to attend to mathematics itself in a classroom setting, &c. To resolve this concern, I point to our ability to engage with mathematics. We have the ability to discover new theorems, and to discuss mathematics. As such, mathematical progress is possible. We make mathematical progress by learning new things about it. Therefore, we can successfully engage with it (adapted from Lo, 2025b). Therefore, we are genuinely engaging with the axiom of choice in the classroom. Therefore, we can attend to mathematics.

Now that we have established what we are attending to, and that we can be successful when attending to mathematics, what happens when we do? Some claim that mathematics is interesting, fascinating, stress-inducing, complex, unintelligible, &c. The average Singaporean would probably cry if you told them to solve some complex integrals involving matrices, and that their grades depended on it. The point here is that Singaporeans are not responding to mathematics itself, but the stress caused by it, and the pressure imposed by our grades being held hostage. These are irrelevant responses to mathematics, and they are not attending to mathematics for its own sake. In contrast, if one were attending to mathematics for its own sake, and then came away with a sense of wonder or a new perspective, would this be an aesthetic experience? I argue that it is, and that this new perspective is a fitting response to the final value that mathematics possesses. One might disagree, arguing that the mathematician is engaging with mathematics with some ulterior motive, i.e. for the sake of their own pleasure—for new perspectives. In this scenario, I fail to see the force of this objection, as the mathematician could easily engage with mathematics with no preconceived end goal, or with any ulterior motives. That is, one can choose to engage with mathematics for its own sake, because they want to, and thus attain an aesthetic experience. Note that Stecker's definition does not require us to have a specific response for such an experience to arise. We need not feel pleasure, or pain, or any strong response. A mundane response would suffice (adapted from Lo, 2025a).

Given the aesthetic experiences that mathematics provides, what does the former say about the value the latter has? Recall my argument for mathematics' independent existence from us. If objects with the property "necessarily true" are valuable in virtue of this property, said objects must be intrinsically valuable. Recall Korsgaard's intrinsic/extrinsic distinction. This distinction is about where the value of an object resides. Since the property of necessarily true resides in mathematics itself, it is intrinsically valuable. Likewise, Korsgaard asserts that an object has final value if we value it for its own sake. Thus, mathematics has final value because it provides aesthetic experiences, as the reason we attend to it for its own sake is because of its final value. Yes, we could attend to something for no real reason, but I argue that we do act with reason when we seek out aesthetic experiences. Consider the Riemann Hypothesis. It is an example of that which has no clear application to the real world. This narrows down our motivations to attend to it. We either attend to it for no good reason, for our own pleasure, or because we find it valuable and fascinating. This tracks with those who say that they pursue pure mathematics because it is intellectually challenging, or that it is aesthetically pleasing. Thus, we attend to pure mathematics for its own sake, because we find it valuable. Thus, p iff q; aesthetic experiences arise from mathematics if and only if it has final value. I acknowledge that I could make my claim more precise by arguing that mathematics is beautiful, where beauty is some aesthetic property, and that gives us reason to value something for its own sake. One could then object to this claim, arguing that we attend to mathematics for the sake of beauty, and not for the sake of mathematics itself. One could counter this objection by arguing that we genuinely attend to objects for their own sake, because they are beautiful. Beauty is the motivating reason as to why we fittingly attend to an object. I am silent about the existence of aesthetic properties—I refer to Stecker's discussion of aesthetic properties (see p. 34 for more). In short, Stecker argues that discussions in reference to aesthetic experiences wholly subsumes discussions of aesthetic properties (p. 35). Thus, we need only speak of aesthetic experiences. Thus, I am satisfied with simply arguing that we act on good reason when we attend to objects for its own sake, without specifying precisely what these reasons are (adapted from Lo, 2025a).

Some would object to my argument, claiming that intrinsic and final value does not exist. They might argue that because aesthetic experiences necessarily involve observers, my argument is observer-dependent. Therefore, the value that mathematics provides only arises because of our observation. Without us, mathematics would not have its supposed final value. Accordingly, they would claim that mathematics only has extrinsic and instrumental value. They might justify this by discussing the effectiveness of mathematics in its description of the natural world, i.e. that it features in physics, engineering, applied mathematics, &c. They would claim that applied mathematics is valuable because it is of use to us. They would argue that pure mathematics has no value, and therefore is a worthless pursuit. Accordingly, they assert that pure mathematics will never give rise to aesthetic experiences because it has no value. I find this argument strange. Recall our discussion of intrinsic value and the isolation test. I have shown that mathematics' capacity to give rise to aesthetic experiences persists in a world without observers. Thus, I find this objection to be overtly observer-centric. Furthermore, aesthetic experiences are subjective. If one believes that an object has no value, they will not have aesthetic experiences (because they are blinded by their presuppositions, or because they will never attend to an object for its own sake, given their beliefs). Instead of resolving this philosophical gridlock by discussing our differing intuitions about the value of mathematics, I point to historic discussions in ethics and value theory. Kant argues that goodwill is the only intrinsic goodness (Korsgaard, 1983, p. 178). Accordingly, this objection would need to respond to Kant's and all other frameworks which argue for the existence of at least one intrinsic or final value. Additionally, this objection would need to contend with the branches of pure mathematics that do not make any reference to the natural world, and thus grapple with the areas where mathematics is not effective in the way in which they desire. Thus, I argue that it is not a problem for my argument to be observer-centric in the way that it is.

A quick aside—aesthetic experiences provide us with a way of discussing the acceptability of proofs. During its initial publication, some argue that computerised proofs, such as the Four Colour Theorem, are unacceptable as we are unable to verify the validity of the proof (by hand, pre-Coq; Tymoczko, 1979, p. 70). The proof only attained its status of acceptance after we discovered new methods of validation which demonstrated that the proof is free of computational error (via the Coq proof checking system; Gonthier, 2005, p. 2). With the benefit of hindsight, I argue that we desire insight from our proofs. That necessitates us knowing the contents of the proof itself. Computerised proofs which are nonsurveyable (i.e. not comprehendable in its entirety; Tymoczko, 1979, p. 59) do not generate insight. Thus, we would like to attend to the contents of the proof. Thus, proofs provide aesthetic experiences iff it is surveyable. Our aesthetic experience is an indicator for the acceptability of a proof. Likewise, the transferability (i.e. "that a relevant expert will become convinced of the truth of the conclusion of the proof just by consideration of each of the steps in the proof"; Easwaran, 2008, p. 343; see Easwaran, 2011, 2014 for a sustained discussion of mathematical knowledge) of proofs is desirable. If we are unable to know the contents of the proof, the proof itself is necessarily not transferable, and will also fail to generate aesthetic experiences (adapted from Lo, 2024). Thus, proofs provide aesthetic experiences iff it is transferable.

To conclude, I note the limitations of my argument. I have supplied a nonconstructive "proof" of mathematics' final and intrinsic value. Like Stecker, I note that we need not decide on the value mathematics has to speak about its aesthetic experiences, nor decide on whether it has value or not. Nevertheless, my argument would have more force if we were to make precise mathematics' final value. However, given the lack of consensus on what this final value might be (in aesthetics and elsewhere), I remain silent on the matter. Furthermore, I have not made precise what are our aesthetic experiences, and what features we attend to when we attend to e.g. the Riemann Hypothesis. Perhaps, mathematics has some advantages over other objects of appreciation, such as classical music and biodiversity, as mathematical properties have clear, rigorous definitions. However, I made no reference to such properties as I find it strange to claim that we find specific problems and theorems interesting because of their specific features and properties. While such properties might captivate us, or be interesting in its own right, I find it more convincing to claim that mathematics itself fascinates us—that all of mathematics is our object of aesthetic appreciation, and we choose to engage with specific parts of it. Regardless, despite these limitations, my argument succeeds as my claims are sufficiently justified. I have also briefly shown what we learn from our aesthetic experiences, as aesthetic experiences are evidence for mathematics' final value, and its acceptability.

References

Easwaran, K. (2008, November 6). Probabilistic Proofs and Transferability. *Philosophia Mathematica, 17*(3), pp. 341–362. https://doi.org/10.1093/philmat/nkn032

Easwaran, K. (2011). *Testimony and Autonomy in Mathematics*. Unpublished draft.

Easwaran, K. (2014, September 1 & 2016, December). Dr. Truthlove, or How I Learned to Stop Worrying and Love Bayesian Probabilities. *Noûs, 50*(4), pp. 816-853. https://www.jstor.org/stable/26631430

Gonthier, G (2005). A computer-checked proof of the Four Colour Theorem. https://api.semanticscholar.org/CorpusID:18529792

Korsgaard, C. M. (1983). Two Distinctions in Goodness. *The Philosophical Review*, *92*(2), 169–195. https://doi.org/10.2307/2184924

Lo, MCJ. (2024). *Mathematics and transferable knowledge* [Undergraduate essay]. For the partial competion of HY3018 Epistemology. Nanyang Technology University.

Lo, MCJ. (2025). *Aesthetic experiences as evidence of biodiversity's final value* [Undergraduate essay]. For the partial completion of HY4131 Ethics of Climate Change (Prof. Conor McHugh). Nanyang Technological University.

Lo, MCJ. (2025). *You, me, and mathematical structures* [Undergraduate essay]. For the partial completion of HY4118 Independent Study I: Philosophy of Mathematics (Prof. Eugene Chua). Nanyang Technological University.

Linnebo, O. (2017). *Philosophy of Mathematics*. Princeton University Press. ISBN: 9780691161402

Shapiro, S. (1997). *Philosophy Of Mathematics: Structure And Ontology*. Oxford University Press. ISBN: 0195094522

Shapiro, S. (2000). *Thinking about Mathematics: The Philosophy of Mathematics*. Oxford University Press. ISBN: 0192893068

Stecker, R. (2019). *Intersections of Value: Art, Nature, and the Everyday*. United Kingdom: Oxford. https://doi.org/10.1093/oso/9780198789956.001.0001

Tymoczko, T (1979). The Four-color Problem and Its Philosophical Significance. *The Journal of Philosophy 76*(2), pp. 57-83. https://doi.org/10.2307/2025976